The above procedure is used by Auld[5] in the solution for magnetostatic modes of spheres and is fully described in Lax and Button.[6] In Schlömann's (10), the magnetic field h is not a source term. It must have the same spatial variation as α, since both h and α are simply the variables in a homogeneous set of equations. If h has an exponential decay in the z-direction, so will α. That is, the solution will consist entirely of the low-k mode. If there is to be any high-k propagating spin wave in the solution for α, there must be a high-k propagating wave in h. Obviously, the unperturbed field of a conductor will not contain a short-wavelength variation. The boundary value problem must be solved to determine whether such a component is present in the YIG.

We believe that Schlömann's (12) is not an inhomogeneous equation and that the distributed source interpretation is not correct. The calculations which follow his (12) are, however, essentially correct because the excitation is not calculated from the unperturbed (without YIG) magnetic field.

Incorrect interpretation of Schlömann's (10) leads to serious errors when the frequency is such that short wavelength spin waves are allowed by the dispersion relation. Under these conditions, the low-k (electromagnetic) solution is also allowed and the h term in Schlömann's (10) matches the unperturbed field almost exactly. ("Unperturbed" means YIG dielectric constant assumed but magnetic dipoles ignored.) Our boundary value solutions show that very little spin wave excitation is obtained, and we think that it makes little difference how many conductors are used or whether they are flat or round.

To date, the only effective linear method of coupling an electromagnetic field to high-k, propagating spin waves is, as predicted by Schlömann, to couple at a point where the spin-wavelength approaches the electromagnetic wavelength. By shaping the dc magnetic field, one can slide along the dispersion relation to the high-k region. Under these conditions, spin-phonon coupling is unavoidable and spin wave defocusing can become a serious problem.

Our calculations are lengthy and not suitable for this published correspondence. They will be supplied to interested workers.

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Messrs. Kaufman and Soohoo[7]

The idea of exciting spin waves in a material by excitation of a specimen with a field that extends only a short distance into the material was proposed before us by Lüthi.[8] In our paper[3] we describe a possible method of accomplishing this by using a fine wire. LaRosa and Vasile maintain that this scheme (and, by inference, any similar scheme) is not valid, since the magnetization must always have the same periodicity as the applied RF h-field; and that the RF h-field must have the same spatial variation as the magnetization.

It should be pointed out here that the excitation of a standing spin wave in a film, first demonstrated by Seavey and Tannenwald,[9] is accomplished with an RF magnetic field distribution that is essentially of constant amplitude throughout the film, yet the spin wave local amplitude varies in the manner of a standing wave distribution. Accordingly, it is not necessary to have the same spatial distribution for RF field as for magnetization.

LaRosa and Vasile state that Schlömann's (12)[4] is not an inhomogeneous equation. This equation is based on the equation of motion of the magnetization,

$$\frac{\partial \boldsymbol{M}}{\partial t} = \gamma(\boldsymbol{M} \times \boldsymbol{H}) + \gamma H_{ex}a^2 \frac{\boldsymbol{M} \times \nabla^2 \boldsymbol{M}}{|\boldsymbol{M}|}. \quad (1)$$

For the infinite medium, assuming fields $\boldsymbol{H} = \hat{a}_x h_x + \hat{a}_y h_y + \hat{a}_z H_0$ and magnetization $\boldsymbol{M} = \hat{a}_x m_x + \hat{a}_y m_y + \hat{a}_z M_0$, where all RF terms vary as exp $(j\omega t)$, this equation is

$$\frac{\partial^2 m_0}{\partial z^2} + \left(\frac{-\gamma H_0 - \omega}{\gamma H_{ex}a^2}\right) m_0 = -\frac{M_0}{H_{ex}a^2} h_0. \quad (2)$$

Here, following Schlömann,[4] $h_0 = h_x + jh_y$; $m_0 = m_x + jm_y$. Neither the spatial distribution of h_0 nor that of m_0 have been specified. As usual, the solution to the homogeneous portion of this equation results in $m_0 = A \exp(jkz) + B \exp(-jkz)$, where $k = [(-\gamma H_0 - \omega)/\gamma H_{ex}a^2]^{1/2}$ is the usual dispersion relation for z-directed spin waves. (Here only variation with z was assumed.) Equation (2) above is therefore clearly inhomogeneous; the h_0 allowing for additional solutions. In these, the A and B coefficients are now not constants where h_0 is nonzero, but they allow for "normal mode" propagation of A, B where $h_0 = 0$. The scheme is analogous to the piezoelectric transducer.

Since this analysis is based solely on the equation of motion (1), it is not completely correct; for a more rigorous solution would also include Maxwell's equations. However, since the exchange power content of a spin wave far exceeds that of the usual Poynting vector power,[10] this treatment should be fairly accurate.

It is of interest to mention here that an earlier analysis[11] states the relation between $\boldsymbol{h}_k$ and $\boldsymbol{m}_k$ of a spin wave, based on Maxwell's equations. In this normal mode analysis, we find

$$\boldsymbol{h}_k = \frac{4\pi(\omega^2\epsilon_e/c^2)\boldsymbol{m}_k - 4\pi \boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{m}_k)}{k^2 - \omega^2\epsilon_e/c^2}. \quad (3)$$

Curiously, $\boldsymbol{h}_k$ and $\boldsymbol{m}_k$ here do not have the same spatial distribution, because of the presence of the $\boldsymbol{k}\cdot\boldsymbol{m}_k$ term.

In conclusion, we wish to state that while our transmission line one-dimensional analysis is a simplification over the actual physical picture, we feel that this analysis is still correct in principle, in the light of the arguments presented here.

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1) Our comments apply to homogeneous magnetic insulators. Spin waves are generated in thin films because of one or more of the following complications: finite conductivity, variation of $4\pi M_s$ with depth, surface pinning (which prevents uniform precession), and small sample size in direction of propagation.

2) We are not sufficiently familiar with the piezoelectric transducer analogy to judge its relevance.

3) We still believe that Maxwell's equations solved simultaneously with the equations of motion yield a homogeneous set of equations in $\bar{m}$, $\bar{h}$ or $\bar{e}$. There are no sources distributed in the volume of the YIG. There are no perturbations in the YIG medium which could serve as sources via perturbation theory.

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Reflection Measurements with Broadband Frequency Modulation Using Long Transmission Lines

This correspondence describes some applications of broadband frequency modulation for measuring reflections on moderately long transmission lines. As known from earlier publications on this subject [1], [2], and from FM-radar techniques, a frequency modulated wave train from a sweep generator is fed into a transmission line. A part of the energy is scattered back by reflections produced on the line or at the end of it. A detector conveniently coupled to the line near the generator provides for mixing of transmitted and scattered wave amplitudes, thus generating an intermediate frequency signal (generally 0.1 . . . 15 kHz) which can be processed by audio frequency techniques.

Ideally, the following equation holds for this audio signal:

$$V_d = \sum_n kr_n \cos\left(\frac{4\pi d_n f_m \Delta F}{v} t - \phi_n\right) \quad (1)$$

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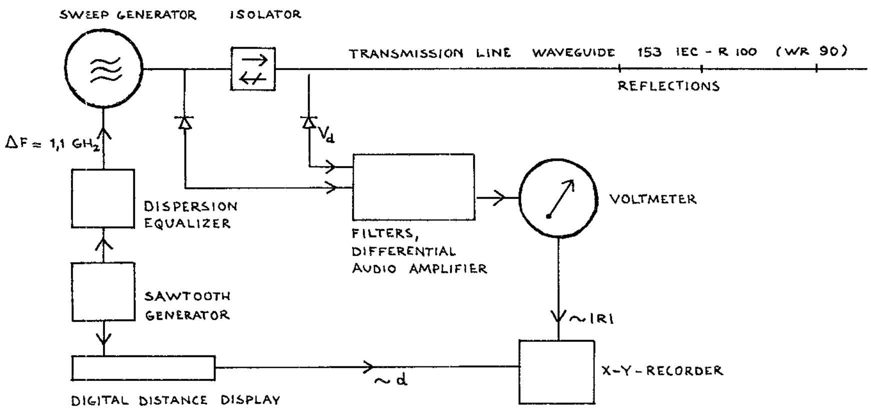

Fig. 1. Block diagram of the distance measuring system.

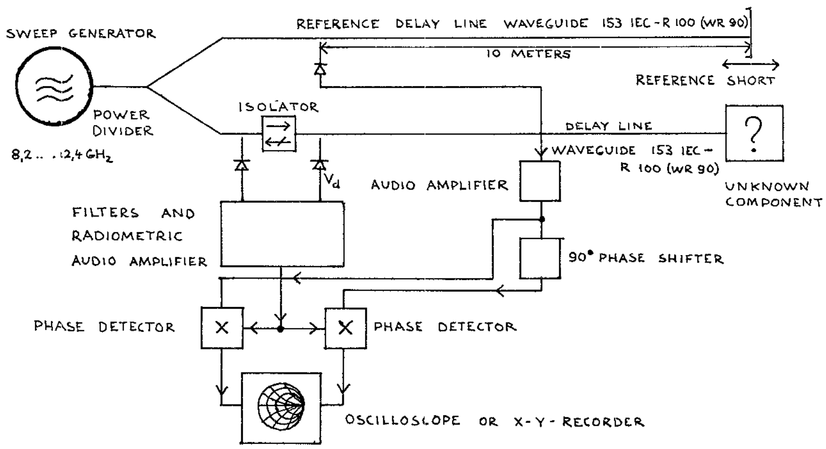

Fig. 2. Block diagram of the Smith chart plotter.

where V_d is the output voltage of the detector, k is a constant, r_n the magnitude of the nth reflection, d_n the distance of the nth reflection, and ϕ_n the phase angle of the nth reflection. f_m is the sawtooth modulation frequency, ΔF the modulation bandwidth, and v the velocity of propagation on the line (v is constant for coaxial lines, but frequency dependent in the case of waveguides; (1) holds, however, for both cases).

Two promising applications of the foregoing principle were developed in detail; the first measuring distance and magnitude of a number of reflections on a transmission line (e.g., an antenna feeder in a microwave relay station), the second measuring magnitude and phase angle of the voltage reflection coefficient of microwave components placed at the end of a delay line.

For the first case an experimental X-band version was built (see Fig. 1). This system proved to have many advantages over a pulse radar of similar performance. It was designed to plot directly reflection magnitude vs. distance. A sensitivity of -70 dB (equivalent to $r=0.0003$ or VSWR 1.0006) referred to a short circuit and an accuracy of ± 1 dB for the magnitude of a reflection and of ± 3 cm for the distance of a reflection were obtained. The useful range of the experimental system was one to twenty meters; however, it could easily be adapted for longer feeder waveguide runs.

For the second case a broadband impedance plotting system was developed, and equally, an experimental X-band version was built (see Fig. 2). This system operates over the whole X-band presenting an oscilloscope or an x-y-ink recorder display of the complex voltage reflection coefficient of an unknown component. The experimental system reached a sensitivity of -60 dB (equivalent to $r=0.001$ or VSWR 1.002) referred to a short circuit and an accuracy of ± 1 dB for the magnitude and of $\pm 5°$ for the phase angle, corresponding to $\pm 0.12r$ within the useful range of $0.001 \leq r \leq 1$. In contrast to earlier developments [1], [2], this system presents the measured impedance (or admittance) directly in a Smith chart display and extends the measurement range about an order of magnitude to smaller reflections.

In both applications a successful attempt has been made to reduce the complexity of the microwave part of the system to a minimum by processing the signals in the audio range. This makes the system easily adaptable to various frequency ranges and waveguide bands.

More detailed analysis is given by Mahle [3].

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Upper and Lower Bounds on the Characteristic Impedance of TEM Mode Transmission Lines

Introduction

A numerical method of obtaining the characteristic impedance of TEM mode transmission lines, which has been described by several authors [1], [2], has the major disadvantage of giving little indication of the resulting error. This correspondence shows how it is possible to extend the finite difference solution of Laplace's Equation to extract an upper and lower bound on the exact solution by using variational formulas. Examples illustrate the high accuracy of solutions obtained with the aid of a digital computer which has been programmed not only to set up and solve the Laplace finite difference equations by systematic overrelaxation but also, at the same time, to compute an upper and a lower bound on the exact solution. Although the method has been used in conjunction with a finite difference solution of Laplace's Equation it can also be used in conjuntion with the Rayleigh Ritz method.

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